

HONORS REVIEW

1) Multiplying / Dividing by NEGATIVE REVERSES SIGN

2) SIGN GRAPH

3 a) $(-\infty, 2\frac{1}{2}]$ b) $(-\infty, \infty)$ c) $(-\infty, -3) \cup (\frac{1}{2}, \infty)$

d) $(-\infty, 3) \cup [7, \infty)$

4) $\frac{y_2 - y_1}{x_2 - x_1}$

5) POSITIVE SLOPE RISES from left to right
NEGATIVE SLOPE FALLS from left to right

6) $y - y_1 = m(x - x_1)$

11) a) $y = 1$ b) $x = -4$ c) $x = -8$

7) $y = mx + b$

d) $y = \frac{1}{4}x + \frac{13}{4}$

8) $Ax + By = C$

$x - 4y = -13$

9) Same Slope

e) $y = \frac{1}{2}x - \frac{5}{2}$

10) OPPOSITE RECIPROALS

$x - 2y = 5$

12) a) $y = x^3 + 1$

14) a) Y-AXIS ONLY

b) $y = x^3 - 7$

b) X-AXIS

c) $y = (x + 3)^3$

Y-AXIS

d) $y = (x - 9)^3$

ORIGIN

e) $y = 2x^3$

15) SET of ORDERED PAIRS where
EVERY x HAS ONLY 1 y value

f) $y = \frac{1}{2}x^3$

16) X-VALUES

g) $y = (-x)^3$

17) Y-VALUES

h) $y = -x^3$

18) SWITCH x AND y

13) a) Replace y with $-y$

19) a) FUNCTION with more
than one part

b) Replace x with $-x$

b) $[x] = y$

c) Replace x with $-x$

c) $y = x$

AND
Replace y with $-y$

20) a) $D: [-5, 5]$ $R: [0, 5]$
 b) $D: (-\infty, \infty)$ $R: [-4, \infty)$

c) $(-\infty, -7) \cup (-7, \infty)$
 $(-\infty, 0) \cup (0, \infty)$

21) a $5x^2 + 12$

b $15/2$

c $5x^2 + 10xh + 5h^2 - 2x - 2h + 8$

d $20x^2 + 76x + 80$

22) $f(-5) = -5$ $f(5) = -22$

23) $y = \frac{1}{3}x + \frac{4}{3}$

24) $y - k = a(x - h)^2$
 $x = h$
 (h, k)

25) $a > 0$ up $a < 0$ down

26) $|a| > 1$ NARROW
 $|a| < 1$ WIDE

27) $y - 3 = 2(x - 1)^2$
 $x = 1$ $(1, 3)$

OPENS UP NARROW
 NO x -INTERCEPTS
 $(0, 5)$

28) $2\frac{1}{2}$ SECONDS 200 ft

29) $f(k) = \text{REMAINDER OF } f(x) \div (x - k)$

30) YES $R = 0$

31) NO $R = 84$

32) 1 POSITIVE 2 or 0 NEGATIVE

33) $f(2) = -2$ $f(3) = -1$

34) a) ALL + 4 1 5 12

b) ALTERNATES 1 -1 1 -2 7

35) $2(x - 2)(x - 3i)(x + 3i)$

36) a) $\pm 1 \pm 2 \pm \frac{1}{2} \pm \frac{1}{4}$

b) $1 - i$ $1 + i$ $-1/4$
 $(x - 1 + i)(x - 1 - i)(4x + 1)$

37) Opposite directions

38) SAME directions

39) TANGENCY

40) FLATTENS OUT

41) LINE GRAPH APPROACHES

b) SET DENOMINATOR = 0

* a) Compare degree
 of NUMERATOR &
 DENOMINATOR

$y = 0$ $y = \frac{a}{b}$

c) \div AND FIND QUOTIENT

42 a) $x = 2$ $x = 4$
 $y = 2$

b) $x = -3$
 $y = x - 3$

43) $\log_a y = x$

44) 10 e

45) -

46) $1 + 2 \log_7 x - \frac{1}{3} \log_7 y$

47) a $1/5$

b) $32/7$ d. $4\frac{1}{2}$

c) -0.0803

REVIEW MATERIAL FOR FINAL EXAM HONORS COLLEGE ALGEBRA

SOLVING INEQUALITIES --- Linear, Quadratic, Rational, Absolute Value

- 1) What is the key property to remember when solving inequalities?
- 2) When solving quadratic or rational inequalities what kind of graph must be used?

- 3) Solve the following inequalities; Write your solution in interval notation.

a) $\frac{3x+7}{-5} \leq 4-x$

$$3x+7 \geq -20+5x$$

$$-2x \geq -27$$

$$x \leq 27/2$$

$$(-\infty, 27/2]$$

b) $|4x+3| > -2$

$$(-\infty, \infty)$$

c) $2x^2+5x-1 > 2$

$$2x^2+5x-3 > 0$$

$$(2x-1)(x+3) > 0$$



$$2x-1$$

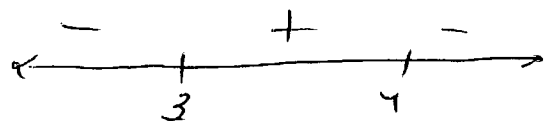
$$x+3$$

$$(-\infty, -3) \cup (1/2, \infty)$$

d) $\frac{x+1}{x-3} \leq 2$

$$\frac{x+1}{x-3} - \frac{2x-6}{x-3} \leq 0$$

$$\frac{-x+7}{x-3} \leq 0$$



$$-x+7$$

$$x-3$$

$$(-\infty, 3) \cup [7, \infty)$$

EQUATIONS OF A LINE

41

- 4) What is the slope formula?
- 5) How does the slope of a line relate to the graph of the line?
- 6) What is the point-slope form of a linear equation?
- 7) What is the slope-intercept form of a linear equation?
- 8) What is the standard form of a linear equation?
- 9) What relationship do the slopes of parallel lines have?
- 10) What relationship do the slopes of perpendicular lines have?
- 11) Write the slope-intercept and standard form of the line that

a) goes through (5,1) and has a slope of 0.

$$\frac{\Delta y}{\Delta x} = \frac{0}{\neq}$$

$$y = 1$$

b) goes through (-4, 13) and is perpendicular to the x-axis

$$x = -4$$

c) has an undefined slope and $f(-8) = 4$

$$\frac{\Delta y}{\Delta x} = \frac{\neq}{0}$$

$$x = -8$$

d) goes through (-1,3) and (3, 4).

$$\frac{4-3}{3-(-1)} = \frac{1}{4}$$

$$y - 3 = \frac{1}{4}(x + 1)$$

$$4y - 12 = x + 1$$

$$x - 4y = -13$$

$$y = \frac{1}{4}x + \frac{13}{4}$$

e) goes through (3, -1) and is parallel to $x - 2y = 4$.

$$-2y = -x + 4$$

$$y = \frac{1}{2}x - 2$$

$$y + 1 = \frac{1}{2}(x - 3)$$

$$2y + 2 = x - 3$$

$$x - 2y = 5$$

$$y = \frac{1}{2}x - \frac{5}{2}$$

FUNCTIONS

H

- 15) What is a function?
- 16) What is the domain of a function?
- 17) What is the range of a function?
- 18) How do you find the inverse of a function?
- 19) What are each of the following special functions?

a) piecewise function

b) greatest integer function

c) identity function

Give the domain and the range for each of the following

a) $y = \sqrt{25 - x^2}$

b) $y = |x^2 + 5| - 9$

c) $y = \frac{3}{7+x}$

$25 - x^2 \geq 0$
 $(5-x)(5+x) \geq 0$
 $x \in [-5, 5]$

$D: (-\infty, \infty)$

$R: [-4, \infty)$

$D: (-\infty, -7) \cup (-7, \infty)$

$R: (-\infty, 0) \cup (0, \infty)$

21) Given $f(x) = 5x^2 - 2x + 8$ and $g(x) = 2x + 4$. Find

a) $(f+g)(x) = 5x^2 + 12$

b) $\left(\frac{f}{g}\right)(-1) = \frac{5(-1)^2 - 2(-1) + 8}{2(-1) + 4} = \frac{5 + 2 + 8}{-2 + 4} = \frac{15}{2}$

c) $f(x+h) = 5(x+h)^2 - 2(x+h) + 8$
 $5x^2 + 10xh + 5h^2 - 2x - 2h + 8$

d) $(f \circ g)(x) = 5(2x+4)^2 - 2(2x+4) + 8$
 $20x^2 + 80x + 80 - 4x - 8 + 8 = 20x^2 + 76x + 80$

22) Given $f(x) = \begin{cases} [x+3] & \text{if } x < -1 \\ -x^2 + 3 & \text{if } -1 \leq x \leq 5 \\ [x-1] & \text{if } x > 5 \end{cases}$

$f(-5) = \frac{-5}{-5+3} = \frac{-5}{-2} = \frac{5}{2}$
 $f(5) = \frac{-22}{(5)^2 + 3} = \frac{-22}{28}$

23) Find the inverse of $y = 3x - 4$.

$x = 3y - 4$
 $3y = x + 4$
 $y = \frac{1}{3}x + \frac{4}{3}$

QUADRATIC FUNCTIONS

H

- 26) What is the standard (h,k) form for a quadratic function?
 24) What is the axis of symmetry and vertex for this function?

$$y - k = a(x - h)^2$$

Axis of Symmetry $x = h$

Vertex (h, k)

- 27) What determines which way a parabola opens?

$a > 0$ up

$a < 0$ down

- 28) What determines how wide or narrow a parabola is?

$|a| > 1$ NARROW

$|a| < 1$ WIDE

- 29) Given the standard (h,k) form, the vertex, axis of symmetry, x-intercepts, y-intercept, direction, scaling, and a sketch of: $f(x) = 2x^2 - 4x + 5$

$$y = 2x^2 - 4x + 5$$

$$y - 5 = 2(x^2 - 2x + 1)$$

$$+ 2 \quad 2(x - 1)^2 + 2$$

$$y - 3 = 2(x - 1)^2$$

X-INTERCEPTS

$$0 - 3 = 2(x - 1)^2$$

$$-3 = 2(x - 1)^2$$

NONE

Axis: $x = 1 \frac{1}{2}$

Vertex $(1, 3) \frac{1}{2}$

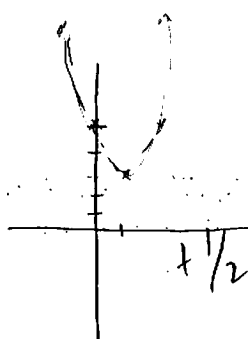
Up $\frac{1}{2}$

NARROW $\frac{1}{2}$

Y-INTERCEPT

$$y = 2(0)^2 - 4(0) + 5$$

$(0, 5)$



- 30) A ball is thrown directly upward from an initial height of 100 feet with an initial velocity of 80 feet per second. Its height is determined by the function $h(t) = -16t^2 + 80t + 100$ where t is the time in seconds after the object is thrown. After how many seconds does the ball reach its maximum height? What is the maximum height of the ball?

$$y = -16t^2 + 80t + 100$$

$$y - 100 = -16(t - 5t + \frac{25}{4})$$

$$y - 200 = -16(t - \frac{5}{2})^2$$

$2 \frac{1}{2}$ SECONDS

200 ft

POLYNOMIAL AND RATIONAL FUNCTIONS

H

29) What is the Remainder Theorem?

30) Use synthetic division to determine if 3 is a zero of the polynomial $f(x) = 2x^5 - 10x^3 - 19x^2 - 45$

$$\begin{array}{r|rrrrrr} 3 & 2 & 0 & -10 & -19 & 0 & -45 \\ & & 6 & 18 & 24 & 15 & 45 \\ \hline & 2 & 6 & 8 & 5 & 15 & 0 \end{array}$$

YES

31) Use synthetic division and the factor theorem to determine if $x + 3$ is a factor of $4x^2 + 2x + 54$.

$$\begin{array}{r|rrr} -3 & 4 & 2 & 54 \\ & & -12 & 30 \\ \hline & 4 & -10 & 84 \end{array}$$

NO

32) According to Descartes' Rule of Signs how many positive and how many negative real zeros does the polynomial function: $f(x) = x^3 - 2x^2 - 13x - 10$ have?

1 positive
2 or 0 negative

$$-x^3 - 2x^2 + 13x - 10$$

33) Use the Intermediate Value Theorem to show there is a real zero between 2 and 3 for the polynomial function $f(x) = 2x^2 - 7x + 4$.

$$f(2) = -2$$

$$f(3) = 1$$

34) Use the Boundedness Theorem to show:

a) there is no real zero greater than 1 for the polynomial function $f(x) = 4x^3 - 3x^2 + 4x + 7$

$$\begin{array}{r|rrrr} 1 & 4 & -3 & 4 & 7 \\ & & 4 & 1 & 5 & 12 \end{array}$$

ALL +

b) there is no real zero less -2 for the polynomial function $f(x) = x^4 + x^3 - x^2 + 3$.

$$\begin{array}{r|rrrrr} -2 & 1 & 1 & -1 & 0 & 3 \\ & & -2 & 2 & -2 & 4 \\ \hline & 1 & -1 & 1 & -2 & 7 \end{array}$$

ALTERNATES

35) Find the polynomial of degree 3 with only real coefficients having zeros of 2 and $3i$; and $f(1) = -20$.

$$\underline{2} (x - 2)(x - 3i)(x + 3i)$$

$$a(-2)(-3i)(3i) = -20$$

$$a(-1)(1 - 9i^2)$$

$$a(-1)(10) = -20$$

$$a = 2$$

36) Given the polynomial function: $f(x) = 4x^3 - 7x^2 + 6x + 2$

a) List all possible rational zeros

$$p: \pm 1 \pm 2 \quad \frac{q}{p} = \pm 1 \pm 2 \pm \frac{1}{2} \pm \frac{1}{4}$$

b) Find all the zeros and corresponding factors given $1 - i$ is a zero.

$$\begin{array}{r} 1-i \overline{) 4 \quad -7 \quad 6 \quad +2} \\ \underline{4 \quad -4i \quad -7-i \quad -2} \\ 1+i \overline{) 4 \quad -3-4i \quad -1-i \quad 0} \\ \underline{4 \quad +4i \quad 1+i} \\ 4 \quad 1 \quad 0 \end{array} \Rightarrow 4x^2 + (-3-4i)x + (-1-i)$$

$$\Rightarrow 4x + 1$$

$$\text{Zeros: } 1-i \quad 1+i \quad -\frac{1}{4}$$

$$\text{Factors: } (x - 1 + i) \quad (x - 1 - i) \quad (4x + 1)$$

GRAPHING POLYNOMIAL AND RATIONAL FUNCTIONS

37) When graphing polynomial functions how do odd degree polynomial functions start and end?

38) When graphing polynomial functions how do even degree polynomial functions start and end?

39) How is even multiplicity shown on a graph of a polynomial function?

40) How is odd multiplicity shown on the graph of a polynomial function?

41) What is an asymptote?

a) How do you find the horizontal asymptotes of a rational function?

b) How do you find the vertical asymptotes of a rational function?

c) How do you find the oblique asymptotes of a rational function?

42) Give the equations of the vertical, horizontal, and oblique asymptotes for:

a) $f(x) = \frac{2x^2 + 5x + 3}{x^2 - 6x + 8}$
 $(x - 2)(x - 4)$

VA: $x = 2 \quad x = 4$

H.A: $y = 2$

b) $f(x) = \frac{x^2 - 1}{x + 3}$

VA: $x = -3$

OA: $y = x - 3$

$$\begin{array}{r} x - 3 \\ x+3 \overline{) x^2 - 1} \\ \underline{x^2 + 3x} \\ -3x - 1 \\ \underline{-3x - 9} \\ 8 \end{array}$$

EXPONENTIAL AND LOGARITHMIC FUNCTIONS

H

44) Give the logarithmic form for the exponential function $y = a^x$: $\log_a y = x$

45) What is the base for each of the following: $\log 7?$ $\ln 7?$
 10 e

46) Complete the properties of logarithmic functions:

a) $\log_a(xy) = \log_a x + \log_a y$

b) $\log_a\left(\frac{x}{y}\right) = \log_a x - \log_a y$

c) $\log_a x^r = r \log_a x$

d) $\log_a b = \frac{\log b}{\log a}$

e) $\log_a a = 1$

f) $\log_a 1 = 0$

47) Solve the following exponential and logarithmic equations:

a) $27^{4x} = 9^{x+1}$

$$(3^3)^{4x} = (3^2)^{x+1}$$

$$12x = 2x + 2$$

$$10x = 2$$

$$x = \frac{1}{5}$$

b) $\log_2 x = 3 + \log_2(x-4)$

$$\log_2 x - \log_2(x-4) = 3$$

$$\log_2 \frac{x}{x-4} = 3$$

$$2^3 = \frac{x}{x-4}$$

$$8 = \frac{x}{x-4}$$

CHECKS
 $x > 0$
 $4\frac{1}{5} > 0$
 \checkmark

$$\begin{aligned} x-4 &> 0 \\ 4\frac{1}{5}-4 &= \\ 4\frac{1}{5} &> 0 \\ \checkmark \end{aligned}$$

$$8x - 32 = x$$

$$7x = 32$$

$$x = \frac{32}{7}$$

$$4\frac{4}{7}$$

c) $6^{1-2x} = 8$

$$\log 6^{1-2x} = \log 8$$

$$(1-2x) \log 6 = \log 8$$

$$\log 6 - 2x \log 6 = \log 8$$

$$-2x \log 6 = \log 8 - \log 6$$

$$x = \frac{\log 8 - \log 6}{-2 \log 6} =$$

d) $\log_6 4x - \log_6(x-3) = \log_6 12$

$$\log_6 \frac{4x}{x-3} = \log_6 12$$

$$\frac{4x}{x-3} = 12$$

$$4x = 12x - 36$$

$$-8x = -36$$

$$x = \frac{36}{8} = \frac{18}{4} = \frac{9}{2} = 4\frac{1}{2}$$

CHECKS

$$4x > 0$$

$$4(\frac{9}{2}) = 18 > 0$$

$$x-3 > 0$$

$$4\frac{1}{2} - 3 = 1\frac{1}{2} > 0$$

$$\frac{\log \frac{4}{3}}{\log \frac{1}{36}}$$

or

$$\frac{\log \frac{3}{4}}{\log 36} = -.0803$$